

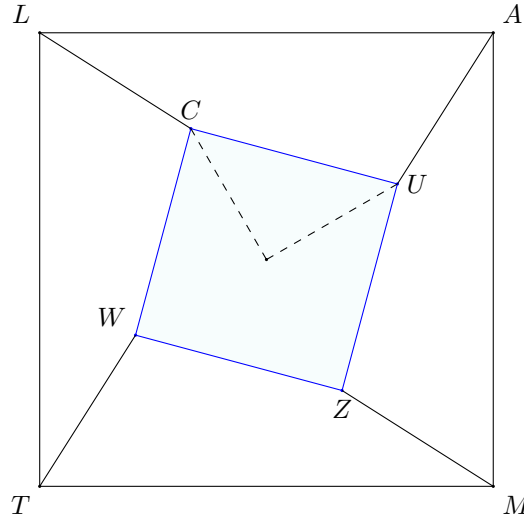
Geometry Round Solutions

May 17, 2026

LAMT 2026

- Let $LAMT$ be a square with center O and side length 12. Points U and C lie inside $LAMT$ so that $UCLA$ is a quadrilateral, $\angle UOC = 90^\circ$, and $OU = OC = 4$. Find the minimum possible area of $UCLA$.

Solution: 28

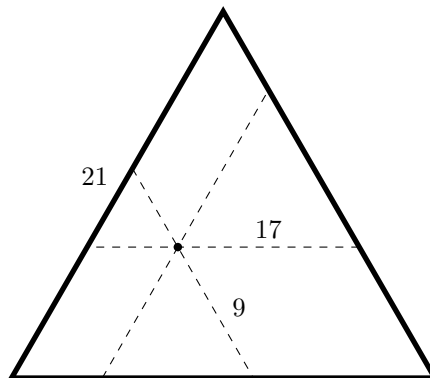


Construct square $CUZW$ centered at O with side length CU . Then by symmetry we have the following

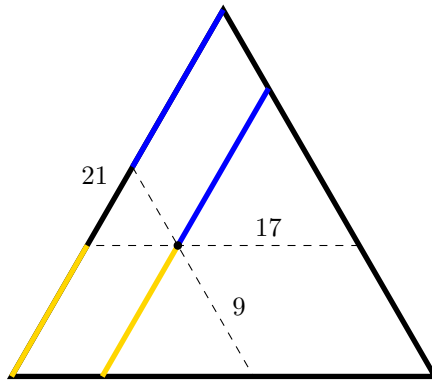
$$UCLA \cong ZUAM \cong WZMT \cong CWTL,$$

so $UCLA$ is $\frac{1}{4}$ of the area inside $LAMT$ but outside $CUZW$. This is just $\frac{12^2 - (4\sqrt{2})^2}{4} = \boxed{28}$. So there is actually only one possible area.

- Aedan has a cookie shaped like an equilateral triangle with side length 21. Aedan makes three cuts through the cookie, each parallel to a different side of the cookie and all three of them passing through a single point. If two of these cuts have lengths 9 and 17, find the length of the third cut.



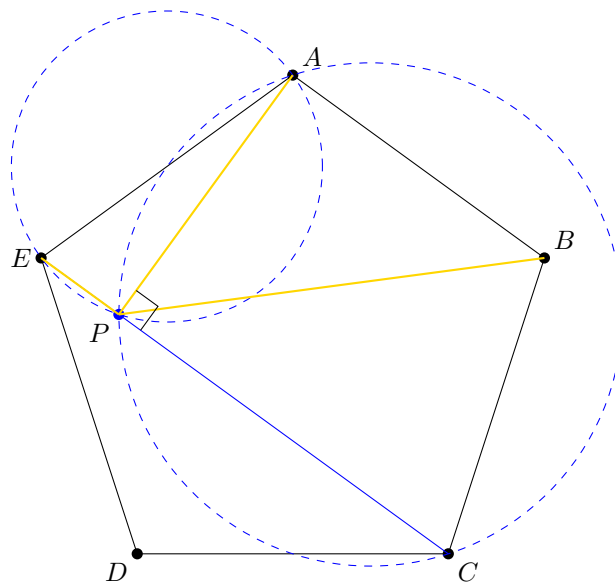
Solution: 16



Note that each of the cuts is cutting an equilateral triangle out of the cookie. From the equilateral triangle with side length 9 we get the blue segments have length $21 - 9 = 12$. Similarly from the equilateral triangle of length 17 we see the gold segments have length 4. Then the answer is $4 + 12 = \boxed{16}$.

3. Let regular pentagon $ABCDE$ have side length 5. The circle with diameter AC intersects the circle with diameter AE at $P \neq A$. Find $PB^2 + PE^2$.

Solution: $\boxed{50}$

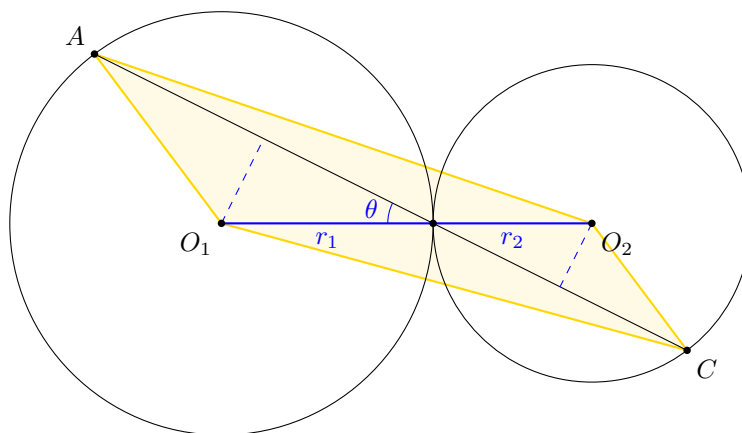


Note that P is the foot from A to EC . Since $ABCE$ is an isosceles trapezoid, $AB \parallel CE$, so $AP \perp AB$. So by the Pythagorean Theorem on triangle BAP we have $PB^2 = PA^2 + AB^2$. Additionally by the Pythagorean Theorem on APE we have

$$PB^2 + PE^2 = AB^2 + PA^2 + PE^2 = AB^2 + AE^2 = 5^2 + 5^2 = \boxed{50}.$$

4. Circles ω_1 , centered at O_1 , and ω_2 , centered at O_2 , are externally tangent at point B . Line segment \overline{AC} passes through B , with $A \neq B$ on ω_1 and $C \neq B$ on ω_2 . Given that the area of quadrilateral O_1CO_2A is 128 and $AC = 32$, find the length of O_1O_2 .

Solution: $\boxed{8\sqrt{5}}$



Let r_1 and r_2 be the radii of ω_1 and ω_2 , respectively. Additionally, let $\theta = \angle O_1BA = \angle O_2BC$. Since the two circles are externally tangent, we need to find $r_1 + r_2$.

We can first find the sum of the heights of O_1 and O_2 to AC , which is just $r_1 \sin \theta + r_2 \sin \theta$. Then the area of the quadrilateral is AC times $\frac{1}{2}$, times this height, which is

$$128 = [AO_2CO_1] = 32 \cdot \frac{1}{2} \cdot (r_1 + r_2) \sin \theta = 16(r_1 + r_2) \sin \theta.$$

Additionally we have $AB = 2r_1 \cos \theta$ and $BC = 2r_2 \cos \theta$, so

$$32 = AC = 2(r_1 + r_2) \cos \theta.$$

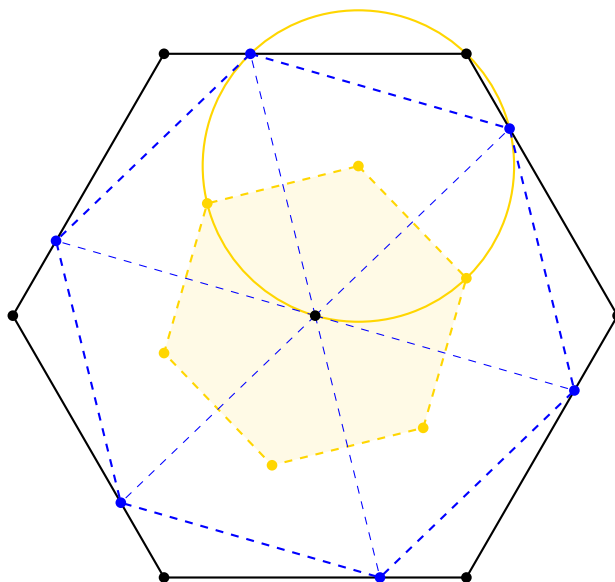
Thus if we let $r_1 + r_2 = x$, we have

$$x \cos \theta = 16, \quad x \sin \theta = 8.$$

From here we can solve $\frac{\sin \theta}{\cos \theta} = \frac{1}{2}$ to get $\sin \theta = \frac{\sqrt{5}}{5}$. This implies that $x = \boxed{8\sqrt{5}}$.

5. Let $A_1A_2A_3A_4A_5A_6$ be a regular hexagon, and let $A_7 = A_1$. For $1 \leq i \leq 6$, point B_i lies on segment $\overline{A_iA_{i+1}}$ such that $A_iB_i = 2$ and $B_iA_{i+1} = 5$. For $1 \leq i \leq 6$ the circumcenter of $B_iA_{i+1}B_{i+1}$ is O_i (where we let $B_7 = B_1$). Find the area of $O_1O_2O_3O_4O_5O_6$.

Solution: $\boxed{\frac{39\sqrt{3}}{2}}$



Let O be the center of $A_1A_2A_3A_4A_5A_6$. This is also the center of hexagon $B_1B_2B_3B_4B_5B_6$. Then note that

$$\angle B_iOB_{i+1} = 60 = 180 - 120 = 180 - \angle B_iA_{i+1}B_{i+1},$$

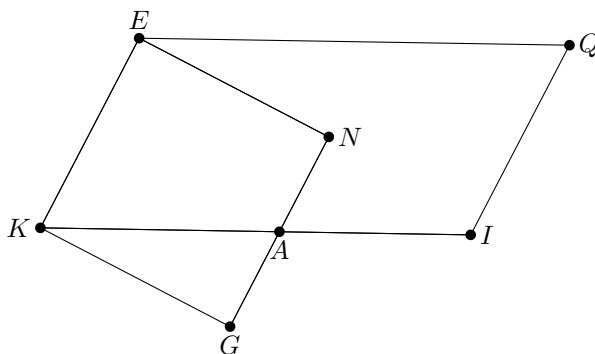
Thus quadrilateral $B_iOB_{i+1}A_{i+1}$ is cyclic for all $1 \leq i \leq 6$. Then the circumcenter of $B_iA_{i+1}B_{i+1}$ is the circumcenter of B_iOB_{i+1} . By Law of Cosines, the side length of $B_1B_2B_3B_4B_5B_6$ is

$$\sqrt{2^2 + 5^2 + 2 \cdot 5} = \sqrt{39}.$$

The side length of $O_1O_2O_3O_4O_5O_6$ is then $\frac{\sqrt{39}}{\sqrt{3}} = \sqrt{13}$. The answer is then

$$\frac{(\sqrt{13})^2 \sqrt{3}}{4} \cdot 6 = \boxed{\frac{39\sqrt{3}}{2}}.$$

6. Parallelogram $KEQI$ satisfies $\frac{KE}{EQ} = \frac{1}{2}$. Square $KENG$ has segment \overline{NG} intersect segment \overline{KI} at point A . Given that the ratio of the area of ANI to the area of ENA is $3 : 5$ and $EA = 5$, find the area of $KEQI$.



Solution: $\boxed{\frac{640}{17}}$

Let $KE = KG = 5x$, which gives $EQ = 10x$. Then the ratio of the areas implies that the ratio of EN to the height from I to AN is $3 : 5$, so the height from I to AN is $3x$.

If we let Z denote the foot from I onto AN , we note that $\triangle KQG \sim \triangle IZA$, and the ratio is $5 : 3$. Thus $KA = 5y$ and $AI = 3y$ for some y . But $KA + AI = KI = 10x$, which implies $y = \frac{5}{4}x$. This gives $KA = \frac{25}{4}x$. Then by the Pythagorean Theorem on KAG , we have

$$GA^2 = KA^2 - KG^2 = \left(\frac{25}{4}x\right)^2 - (5x)^2 = \left(\frac{15}{4}x\right)^2.$$

Then $GN = 5x$, so $AN = 5x - \frac{15}{4}x = \frac{5}{4}x$. By the Pythagorean Theorem on ENA , we get

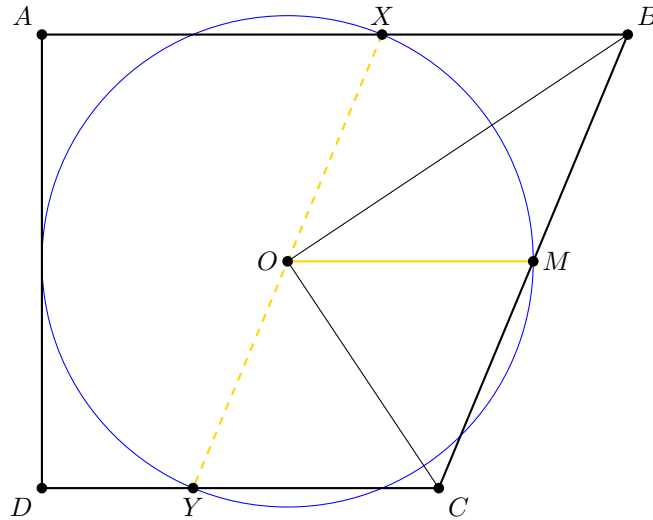
$$5 = EA = \frac{5\sqrt{17}}{4}x \implies x = \frac{4}{\sqrt{17}}.$$

Then note that the area of $KEQI$ is $KE = 5x$ times the height, which is $5x + 3x = 8x$. So the area is $40x^2$. Plugging in, we get an answer of

$$40 \cdot \left(\frac{4}{\sqrt{17}}\right)^2 = \frac{40 \cdot 16}{17} = \boxed{\frac{640}{17}}.$$

7. Let $ABCD$ be a convex quadrilateral with $\angle A = \angle D = 90^\circ$ and $CD < AB = 31$. A circle intersects the midpoint of BC and is tangent to AD . In addition, segments \overline{AB} , \overline{BC} , and \overline{CD} each intersect the circle at 2 points, forming three chords of length 10. Find the area of $ABCD$.

Solution: $\boxed{624}$



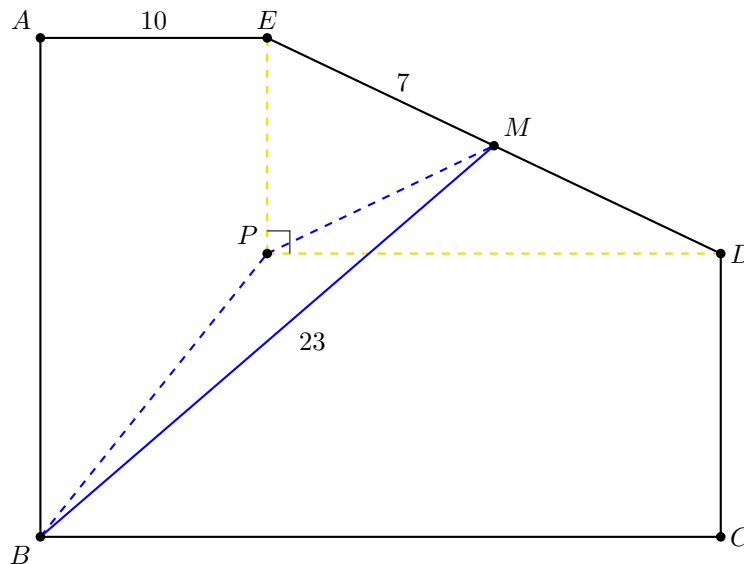
Let the center of the circle be O . Since all the intersections have length 10, O is equidistant to AB , BC , and CD . Thus BO bisects $\angle B$ and CO bisects $\angle C$, and we get $\angle BOC = 90^\circ$.

Let the midpoint of BC be M , the intersection of the circle and AB closer to B be X , and the intersection of the circle and CD closer to D be Y . Then since BOC is a right triangle, and M is the midpoint of the hypotenuse, we have $OM = MB = MC$. But $MB = BX$, so $OXBM$ is a rhombus. Similarly $OMCY$ is a rhombus.

We see that difference between AB and CD is the length equal to 10, since $XBCY$ is a parallelogram. This $CD = 21$. Then the midline of the trapezoid has length $\frac{31+21}{2} = 26$, which is twice the radius of the circle. To finish we can calculate the height using the Pythagorean Theorem as $2\sqrt{13^2 - 5^2} = 24$. The answer is then $24 \cdot 26 = \boxed{624}$.

8. Convex pentagon $ABCDE$ has $\angle A = \angle B = \angle C = 90^\circ$. Let M be the midpoint of ED . Suppose $AE = 10$, $ED = 14$, and $BM = 23$. Find the minimum possible length of CD .

Solution: $\boxed{2\sqrt{39}}$



Let $CD = x$. Construct the point X satisfying $XE \perp EA$ and $XD \perp DC$. Then note that $\angle EXD = 90^\circ$, so $EM = MD = 7$, since M is the circumcenter of XED . Additionally $BX = \sqrt{10^2 + x^2}$. By the triangle

inequality on BXM , we have

$$23 = BM \leq XM + MB = 7 + \sqrt{10^2 + x^2} \implies x \geq \boxed{2\sqrt{39}}.$$

This configuration exists as we can take a $10 \times 2\sqrt{39}$ rectangle and extend the diagonal BX passed X to a point M with $BM = 23$. Then draw the circle centered at M with radius 7 and let it intersect the extensions of the sides of the rectangle at E and D . Then draw points A and C accordingly.

Remark. This problem was first written as an algebra problem. Here is the original statement:

Suppose a and b are complex numbers in the first quadrant satisfying

$$\Re(a) = 10, \quad |a - b| = 14, \quad \text{and} \quad |a + b| = 46.$$

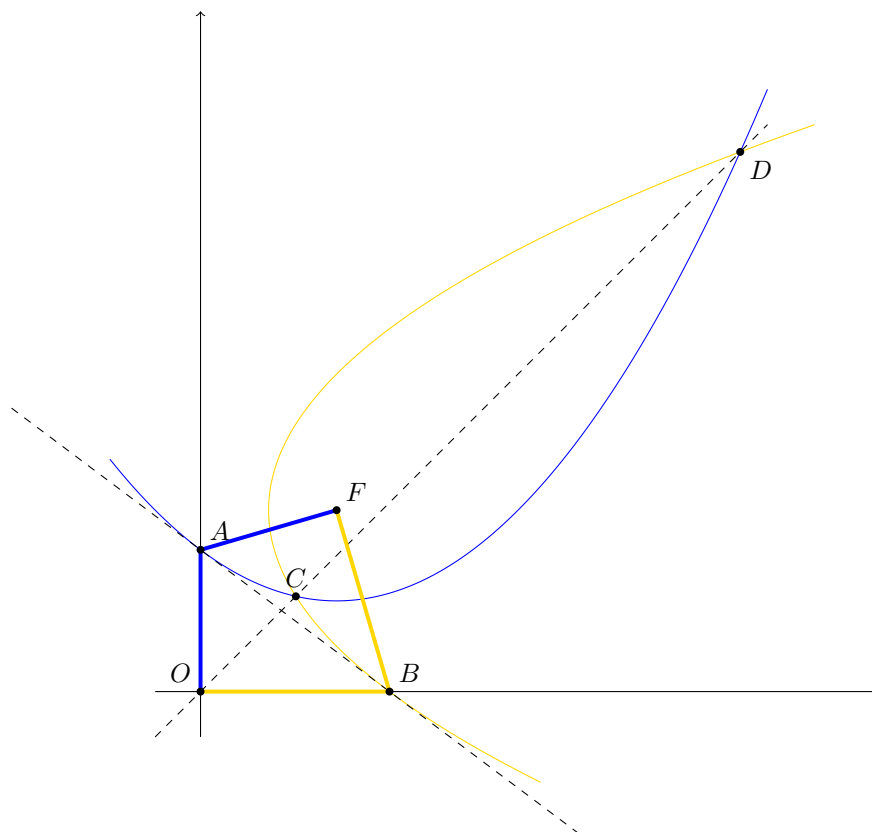
Find the minimum possible value of $\Im(b)$.

(Here $\Re(z)$ denotes the real part of z and $\Im(z)$ denotes the imaginary part of z).

9. Let \mathcal{P}_1 and \mathcal{P}_2 be two parabolas with focus $(6, 8)$ and directrices equal to the x -axis and the y -axis, respectively. The common external tangent of the two parabolas and the line connecting their two points of intersection meet at point X . Find the coordinates of X .

Solution: $\boxed{\left(\frac{25}{7}, \frac{25}{7}\right)}$

Let O be the origin, let F be the point $(6, 8)$, let A be the intersection of \mathcal{P}_1 and the y -axis, and let B be the intersection of \mathcal{P}_2 and the x -axis. Then by the definition of a parabola we have $AO = AF$ and $BO = BF$. Thus we have $\triangle AOB \cong \triangle AFB$.



Now we claim AB is the common tangent of the parabolas. Indeed $\angle FAB = \angle BAO$, which is well-known to mean this is the tangent. We know AB is the perpendicular bisector of FO , so it has equation $3x + 4y = 25$.

Thus we get $FH : HG : GE = 5 : 7 : 2$.

Since AD is tangent to both circles, so

$$\angle ADE = \angle ECD, \quad \angle ADF = \angle ABC \implies \angle BAC + \angle FDE = 180^\circ.$$

This means $AFDE$ is a cyclic quadrilateral. Since $HX \cdot XF = XD^2$, we can write $XD = \sqrt{8x \cdot 3x} = 2\sqrt{6}x$. Then by Power of a Point at X with respect to $(AFDE)$, we get

$$AX \cdot DX = FX \cdot XE \implies AX = 4\sqrt{6}x.$$

By Stewart's Theorem on $\triangle AFE$ with cevian AX , we let $y = 2\sqrt{3}x$ which gives $AF = 4y$, $AE = 3y$. As $\triangle AEF \sim \triangle ABC$, we get the answer is

$$\frac{AE}{EF} = \frac{3\sqrt{3}}{7} = \frac{AB}{BC} \implies BC = \boxed{\frac{14\sqrt{3}}{3}}.$$

11. **[TIEBREAKER]** Let O be the origin, and \mathcal{C} be a unit cube centered at O . A point P is selected uniformly at random from the surface of a unit sphere centered at O . The plane passing through O which is perpendicular to OP intersects \mathcal{C} at a polygon \mathcal{Q} . Estimate the expected value of the length of the longest side of \mathcal{Q} .

Express your answer as a number in base 10 (submissions not in this form will not be accepted). Ties will be broken based on distance to the correct answer.

Solution: $\boxed{1.0975}$